5.1 Lesson

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Lesson Tutorials
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Key Vocabulary
ratio, *p. 164*
rate, *p. 164*
unit rate, *p. 164*
complex fraction, *p. 165*

A **ratio** is a comparison of two quantities using division.

$\frac{3}{4}$, 3 to 4, 3 : 4

A **rate** is a ratio of two quantities with different units.

$\frac{60 \text{ miles}}{2 \text{ hours}}$

A rate with a denominator of 1 is called a **unit rate**.

$\frac{30 \text{ miles}}{1 \text{ hour}}$

EXAMPLE 1 Finding Ratios and Rates

There are 45 males and 60 females in a subway car. The subway car travels 2.5 miles in 5 minutes.

a. Find the ratio of males to females.

b. Find the speed of the subway car.

EXAMPLE 2 Finding a Rate from a Ratio Table

The ratio table shows the costs for different amounts of artificial turf. Find the unit rate in dollars per square foot.

× 4 × 4 × 4

Amount (square feet)	25	100	400	1600
Cost (dollars)	100	400	1600	6400

× 4 × 4 × 4

Remember
The abbreviation ft^2 means *square feet.*

Multi-Language Glossary at BigIdeasMath.com

On Your Own

Now You're Ready
Exercises 11–24

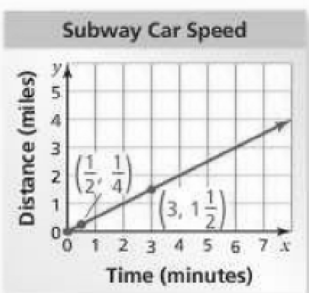
1. In Example 1, find the ratio of females to males.
2. In Example 1, find the ratio of females to total passengers.
3. The ratio table shows the distance that the *International Space Station* travels while orbiting Earth. Find the speed in miles per second.

Time (seconds)	3	6	9	12
Distance (miles)	14.4	28.8	43.2	57.6

A **complex fraction** has at least one fraction in the numerator, denominator, or both. You may need to simplify complex fractions when finding ratios and rates.

EXAMPLE 3 Finding a Rate from a Graph

The graph shows the speed of a subway car. Find the speed in miles per minute. Compare the speed to the speed of the subway car in Example 1.



On Your Own

Now You're Ready
Exercise 28

4. You use the point $\left(3, 1\frac{1}{2}\right)$ to find the speed of the subway car. Does your answer change? Explain your reasoning.

EXAMPLE 4 Solving a Ratio Problem

You mix $\frac{1}{2}$ cup of yellow paint for every $\frac{3}{4}$ cup of blue paint to make 15 cups of green paint. How much yellow paint and blue paint do you use?

Math Practice 1

Analyze Givens

What information is given in the problem? How does this help you know that the ratio table needs a "total" column? Explain.

On Your Own

Now You're Ready
Exercises 33 and 34

5. How much yellow paint and blue paint do you use to make 20 cups of green paint?

5.2 Proportions

Essential Question How can proportions help you decide when things are "fair"?

The Meaning of a Word ● Proportional

When you work toward a goal, your success is usually **proportional** to the amount of work you put in.

An equation stating that two ratios are equal is a **proportion**.

1 ACTIVITY: Determining Proportions

Work with a partner. Tell whether the two ratios are equivalent. If they are not equivalent, change the next day to make the ratios equivalent. Explain your reasoning.

a. On the first day, you pay \$5 for 2 boxes of popcorn. The next day, you pay \$7.50 for 3 boxes.

b. On the first day, it takes you $3\frac{1}{2}$ hours to drive 175 miles. The next day, it takes you 5 hours to drive 200 miles.

COMMON CORE

Proportions

In this lesson, you will

- use equivalent ratios to determine whether two ratios form a proportion.
- use the Cross Products Property to determine whether two ratios form a proportion.

Learning Standard MACC.7.RP.1.2a

c. On the first day, you walk 4 miles and burn 300 calories. The next day, you walk $3\frac{1}{3}$ miles and burn 250 calories.

d. On the first day, you paint 150 square feet in $2\frac{1}{2}$ hours. The next day, you paint 200 square feet in 4 hours.

First Day   Next Day

$$\frac{150\text{ ft}^2}{2\frac{1}{2}\text{ h}} \stackrel{?}{=} \frac{200\text{ ft}^2}{4\text{ h}}$$

5.2 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary
proportion, *p. 172*
proportional, *p. 172*
cross products, *p. 173*

Key Idea

Proportions

Words A **proportion** is an equation stating that two ratios are equivalent. Two quantities that form a proportion are **proportional**.

Numbers $\frac{2}{3} = \frac{4}{6}$ The proportion is read "2 is to 3 as 4 is to 6."

EXAMPLE 1 Determining Whether Ratios Form a Proportion

Tell whether $\frac{6}{4}$ and $\frac{8}{12}$ form a proportion.

Compare the ratios in simplest form.

So, $\frac{6}{4}$ and $\frac{8}{12}$ do *not* form a proportion.

EXAMPLE 2 Determining Whether Two Quantities Are Proportional

Tell whether x and y are proportional.

x	y
$\frac{1}{2}$	3
1	6
$\frac{3}{2}$	9
2	12

Reading
Two quantities that are proportional are in a *proportional relationship*.

On Your Own

Now You're Ready
Exercises 5–14

Tell whether the ratios form a proportion.

1. $\frac{1}{2}, \frac{5}{10}$
2. $\frac{4}{6}, \frac{18}{24}$
3. $\frac{10}{3}, \frac{5}{6}$
4. $\frac{25}{20}, \frac{15}{12}$
5. Tell whether x and y are proportional.

Birdhouses Built, x	1	2	4	6
Nails Used, y	12	24	48	72

Key Ideas

Cross Products

In the proportion $\frac{a}{b} = \frac{c}{d}$, the products $a \cdot d$ and $b \cdot c$ are called **cross products**.

Study Tip

You can use the Multiplication Property of Equality to show that the cross products are equal.

$$\frac{a}{b} = \frac{c}{d}$$

$$\cancel{b}d \cdot \frac{a}{\cancel{b}} = b\cancel{d} \cdot \frac{c}{\cancel{d}}$$

$$ad = bc$$

Cross Products Property

Words The cross products of a proportion are equal.

Numbers

$$\frac{2}{3} = \frac{4}{6}$$

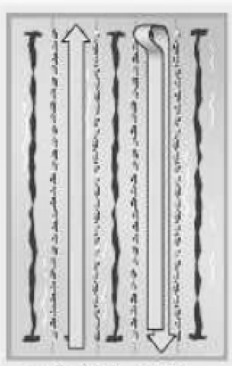
$2 \cdot 6 = 3 \cdot 4$

Algebra

$$\frac{a}{b} = \frac{c}{d}$$

$ad = bc$,
where $b \neq 0$ and $d \neq 0$

EXAMPLE 3 Identifying Proportional Relationships


1 length 1 lap

You swim your first 4 laps in 2.4 minutes. You complete 16 laps in 12 minutes. Is the number of laps proportional to your time?

Method 1: Compare unit rates.

Method 2: Use the Cross Products Property.

On Your Own

Now You're Ready
Exercises 15–20

6. You read the first 20 pages of a book in 25 minutes. You read 36 pages in 45 minutes. Is the number of pages read proportional to your time?

Extension 5.2 Graphing Proportional Relationships

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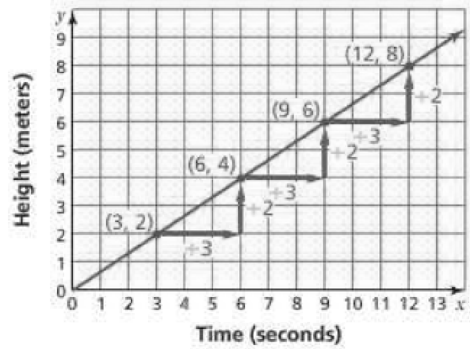
Recall that you can graph the values from a ratio table.

	Time, x (seconds)	Height, y (meters)	
	3	2	
$+3$	6	4	$+2$
$+3$	9	6	$+2$
$+3$	12	8	$+2$



The structure in the ratio table shows why the graph has a constant *rate of change*. You can use the constant rate of change to show that the graph passes through the origin. The graph of every proportional relationship is a line through the origin.

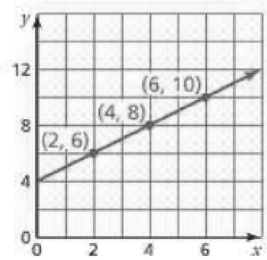
EXAMPLE 1 Determining Whether Two Quantities Are Proportional

Use a graph to tell whether x and y are in a proportional relationship.

a.

x	2	4	6
y	6	8	10

Plot (2, 6), (4, 8), and (6, 10). Draw a line through the points.



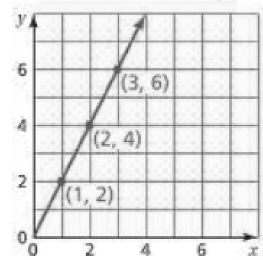
The graph is a line that does not pass through the origin.

∴ So, x and y are not in a proportional relationship.

b.

x	1	2	3
y	2	4	6

Plot (1, 2), (2, 4), and (3, 6). Draw a line through the points.



The graph is a line that passes through the origin.

∴ So, x and y are in a proportional relationship.

COMMON CORE

Proportions

In this extension, you will

- use graphs to determine whether two ratios form a proportion.
- interpret graphs of proportional relationships.

Learning Standards
MACC.7.RP.1.2a
MACC.7.RP.1.2b
MACC.7.RP.1.2d

Practice

Use a graph to tell whether x and y are in a proportional relationship.

1.

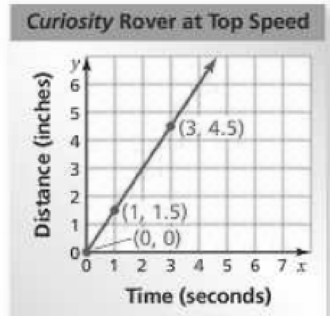
x	1	2	3	4
y	3	4	5	6

2.

x	1	3	5	7
y	0.5	1.5	2.5	3.5

EXAMPLE 2 Interpreting the Graph of a Proportional Relationship

The graph shows that the distance traveled by the Mars rover *Curiosity* is proportional to the time traveled. Interpret each plotted point in the graph.



(0, 0): The rover travels 0 inches in 0 seconds.

(1, 1.5): The rover travels 1.5 inches in 1 second. So, the unit rate is 1.5 inches per second.

(3, 4.5): The rover travels 4.5 inches in 3 seconds. Because the relationship is proportional, you can also use this point to find the unit rate.

$$\frac{4.5 \text{ in.}}{3 \text{ sec}} = \frac{1.5 \text{ in.}}{1 \text{ sec}}, \text{ or } 1.5 \text{ inches per second}$$

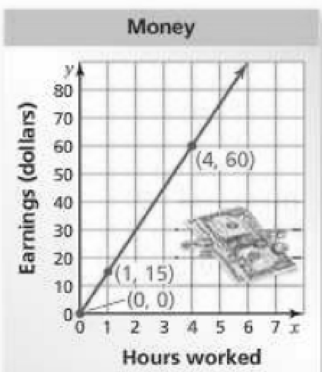
Study Tip

In the graph of a proportional relationship, you can find the unit rate from the point (1, *y*).

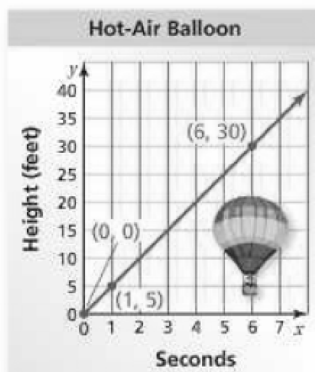
Practice

Interpret each plotted point in the graph of the proportional relationship.

3.



4.



Tell whether *x* and *y* are in a proportional relationship. If so, find the unit rate.

5.

x (hours)	1	4	7	10
y (feet)	5	20	35	50

6. Let *y* be the temperature *x* hours after midnight. The temperature is 60°F at midnight and decreases 2°F every $\frac{1}{2}$ hour.

7. REASONING The graph of a proportional relationship passes through (12, 16) and (1, *y*). Find *y*.

8. MOVIE RENTAL You pay \$1 to rent a movie plus an additional \$0.50 per day until you return the movie. Your friend pays \$1.25 per day to rent a movie.

a. Make tables showing the costs to rent a movie up to 5 days.

b. Which person pays an amount proportional to the number of days rented?

5.3 Lesson

Check It Out
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One way to write a proportion is to use a table.

	Last Month	This Month
Purchase	2 ringtones	3 ringtones
Total Cost	6 dollars	x dollars

Use the columns or the rows to write a proportion.

Use columns:

$$\frac{2 \text{ ringtones}}{6 \text{ dollars}} = \frac{3 \text{ ringtones}}{x \text{ dollars}}$$

Numerators have the same units.

Denominators have the same units.

Use rows:

$$\frac{2 \text{ ringtones}}{3 \text{ ringtones}} = \frac{6 \text{ dollars}}{x \text{ dollars}}$$

The units are the same on each side of the proportion.

EXAMPLE 1 Writing a Proportion

Black Bean Soup

1.5 cups black beans
0.5 cup salsa
2 cups water
1 tomato
2 teaspoons seasoning

A chef increases the amounts of ingredients in a recipe to make a proportional recipe. The new recipe has 6 cups of black beans. Write a proportion that gives the number x of tomatoes in the new recipe.

Organize the information in a table.

	Original Recipe	New Recipe
Black Beans		
Tomatoes		

On Your Own

Now You're Ready
Exercises 8–11

1. Write a different proportion that gives the number x of tomatoes in the new recipe.
2. Write a proportion that gives the amount y of water in the new recipe.

EXAMPLE 2 Solving Proportions Using Mental Math

Solve $\frac{3}{2} = \frac{x}{8}$.

EXAMPLE 3 Solving Proportions Using Mental Math

In Example 1, how many tomatoes are in the new recipe?

Solve the proportion $\frac{1.5}{1} = \frac{6}{x}$.

cups black beans

tomatoes

On Your Own

Now You're Ready
Exercises 16–21

Solve the proportion.

3. $\frac{5}{8} = \frac{20}{d}$ **4.** $\frac{7}{z} = \frac{14}{10}$ **5.** $\frac{21}{24} = \frac{x}{8}$

6. A school has 950 students. The ratio of female students to all students is $\frac{48}{95}$. Write and solve a proportion to find the number f of students who are female.

5.4 Lesson

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Key Idea

Solving Proportions

Method 1 Use mental math. *(Section 5.3)*

Method 2 Use the Multiplication Property of Equality. *(Section 5.4)*

Method 3 Use the Cross Products Property. *(Section 5.4)*

EXAMPLE 1 Solving Proportions Using Multiplication

On Your Own

Now You're Ready
Exercises 4–9

Use multiplication to solve the proportion.

1. $\frac{w}{6} = \frac{6}{9}$ **2.** $\frac{12}{10} = \frac{a}{15}$ **3.** $\frac{y}{6} = \frac{2}{4}$

EXAMPLE 2 Solving Proportions Using the Cross Products Property

Solve each proportion.

a. $\frac{x}{8} = \frac{7}{10}$ **b.** $\frac{9}{y} = \frac{3}{17}$

On Your Own

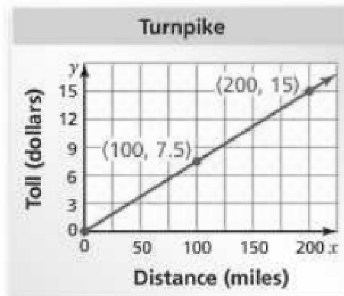
Now You're Ready
Exercises 10–21

Use the Cross Products Property to solve the proportion.

4. $\frac{2}{7} = \frac{x}{28}$ **5.** $\frac{12}{5} = \frac{6}{y}$ **6.** $\frac{40}{z+1} = \frac{15}{6}$

EXAMPLE 3 Real-Life Application

The graph shows the toll y due on a turnpike for driving x miles. Your toll is \$7.50. How many *kilometers* did you drive?



The point (100, 7.5) on the graph shows that the toll is \$7.50 for driving 100 miles. Convert 100 miles to kilometers.

Method 1: Convert using a ratio.

Method 2: Convert using a proportion.

Let x be the number of kilometers equivalent to 100 miles.

On Your Own

Now You're Ready
Exercises 28–30

Write and solve a proportion to complete the statement. Round to the nearest hundredth, if necessary.

7. 7.5 in. ≈ ___ cm   **8.** 100 g ≈ ___ oz

9. 2 L ≈ ___ qt   **10.** 4 m ≈ ___ ft

5.5 Lesson

Check It Out
Lesson Tutorials
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Key Vocabulary
slope, *p. 194*

Study Tip

The slope of a line is the same between any two points on the line because lines have a *constant* rate of change.

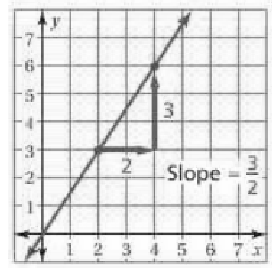
Key Idea

Slope

Slope is the rate of change between any two points on a line. It is a measure of the *steepness* of a line.

To find the slope of a line, find the ratio of the change in y (vertical change) to the change in x (horizontal change).

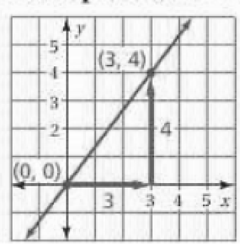
$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$



EXAMPLE 1 Finding Slopes

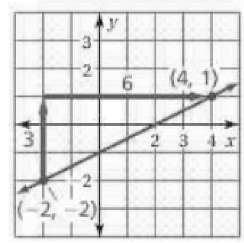
Find the slope of each line.

a.



$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

b.



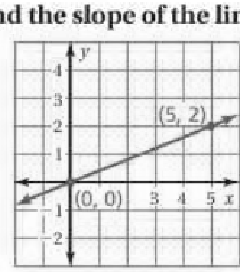
$$\text{slope} = \frac{\text{change in } y}{\text{change in } x}$$

On Your Own

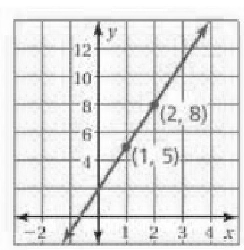
Now You're Ready
Exercises 4–9

Find the slope of the line.

1.



2.



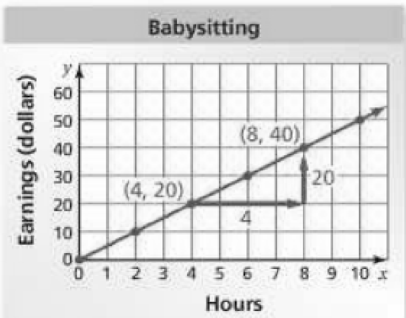
EXAMPLE 2 Interpreting a Slope

The table shows your earnings for babysitting.

a. Graph the data.

b. Find and interpret the slope of the line through the points.

Hours, x	0	2	4	6	8	10
Earnings, y (dollars)	0	10	20	30	40	50



a. Graph the data. Draw a line through the points.

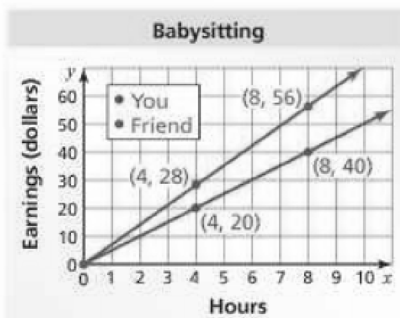
b. Choose any two points to find the slope of the line.

On Your Own

Now You're Ready
Exercises 10 and 11

3. In Example 2, use two other points to find the slope. Does the slope change?

4. The graph shows the amounts you and your friend earn babysitting.



a. Compare the steepness of the lines. What does this mean in the context of the problem?

b. Find and interpret the slope of the blue line.

5.6 Lesson

Check It Out
Lesson Tutorials
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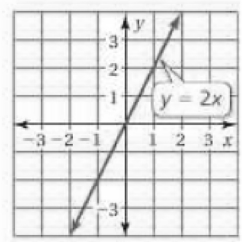
Key Vocabulary
direct variation, *p. 200*
constant of proportionality, *p. 200*

Key Idea

Direct Variation

Words Two quantities x and y show **direct variation** when $y = kx$, where k is a number and $k \neq 0$. The number k is called the **constant of proportionality**.

Graph The graph of $y = kx$ is a line with a slope of k that passes through the origin. So, two quantities that show direct variation are in a proportional relationship.



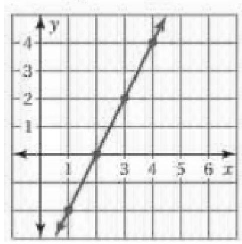
EXAMPLE 1 Identifying Direct Variation

Tell whether x and y show direct variation. Explain your reasoning.

a.

x	1	2	3	4
y	−2	0	2	4

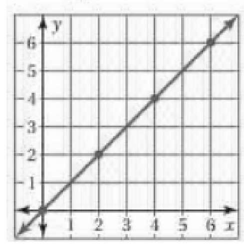
Plot the points. Draw a line through the points.



b.

x	0	2	4	6
y	0	2	4	6

Plot the points. Draw a line through the points.



Study Tip
Other ways to say that x and y show direct variation are "y varies directly with x" and "x and y are directly proportional."

EXAMPLE 2 Identifying Direct Variation

Tell whether x and y show direct variation. Explain your reasoning.

a. $y + 1 = 2x$

b. $\frac{1}{2}y = x$

On Your Own

Now You're Ready
Exercises 6–17

Tell whether x and y show direct variation. Explain your reasoning.

1.

x	y
0	−2
1	1
2	4
3	7

2.

x	y
1	4
2	8
3	12
4	16

3.

x	y
−2	4
−1	2
0	0
1	2

4. $xy = 3$

5. $x = \frac{1}{3}y$

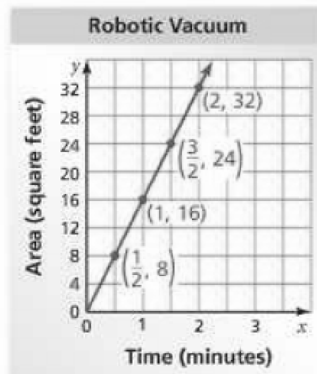
6. $y + 1 = x$

EXAMPLE 3 Real-Life Application

x	y
$\frac{1}{2}$	8
1	16
$\frac{3}{2}$	24
2	32

The table shows the area y (in square feet) that a robotic vacuum cleans in x minutes.

a. Graph the data. Tell whether x and y are directly proportional.



b. Write an equation that represents the line.

c. Use the equation to find the area cleaned in 10 minutes.