Chapter 9
Surface Area and Volume

9.1 Lesson

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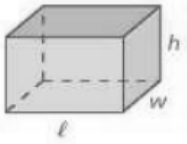
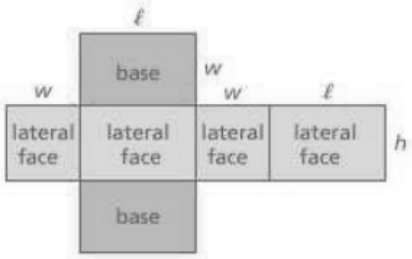
Key Vocabulary
lateral surface area, *p. 358*

Key Idea

Surface Area of a Rectangular Prism

Words The surface area S of a rectangular prism is the sum of the areas of the bases and the lateral faces.

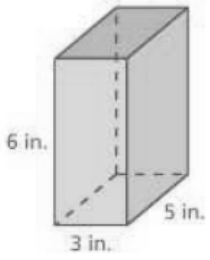




Algebra $S = 2\ell w + 2\ell h + 2wh$

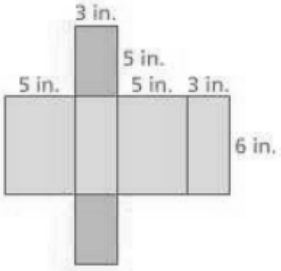
Areas of bases — Areas of lateral faces

EXAMPLE 1 Finding the Surface Area of a Rectangular Prism



Find the surface area of the prism.

Draw a net.



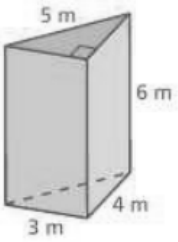
Key Idea

Surface Area of a Prism

The surface area S of any prism is the sum of the areas of the bases and the lateral faces.

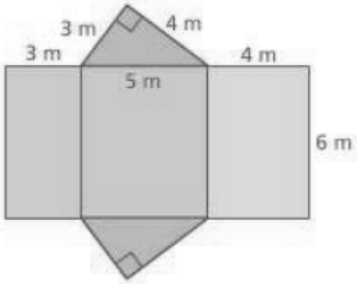
S = areas of bases + areas of lateral faces

EXAMPLE 2 Finding the Surface Area of a Triangular Prism



Find the surface area of the prism.

Draw a net.



Remember

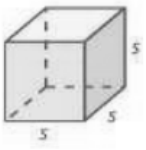
The area A of a triangle with base b and height h is $A = \frac{1}{2}bh$.

Remember

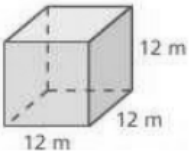
A cube has 6 congruent square faces.

When all the edges of a rectangular prism have the same length s, the rectangular prism is a cube. The formula for the surface area of a cube is

$S = 6s^2$. Formula for surface area of a cube



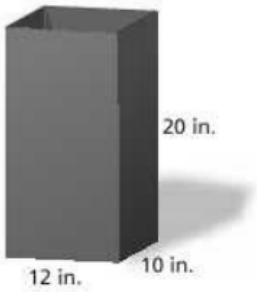
EXAMPLE 3 Finding the Surface Area of a Cube



Find the surface area of the cube.

EXAMPLE 4 Real-Life Application

The outsides of purple traps are coated with glue to catch emerald ash borers. You make your own trap in the shape of a rectangular prism with an open top and bottom. What is the surface area that you need to coat with glue?



Find the lateral surface area.

$S = 2\ell h + 2wh$ ← Do not include the areas of the bases in the formula.

9.2 Lesson

Check It Out
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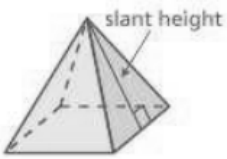
Key Vocabulary
regular pyramid, *p. 364*
slant height, *p. 364*

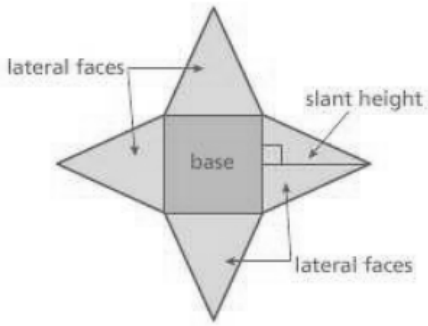
A **regular pyramid** is a pyramid whose base is a regular polygon. The lateral faces are triangles. The height of each triangle is the **slant height** of the pyramid.

Key Idea

Surface Area of a Pyramid

The surface area S of a pyramid is the sum of the areas of the base and the lateral faces.



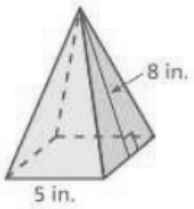


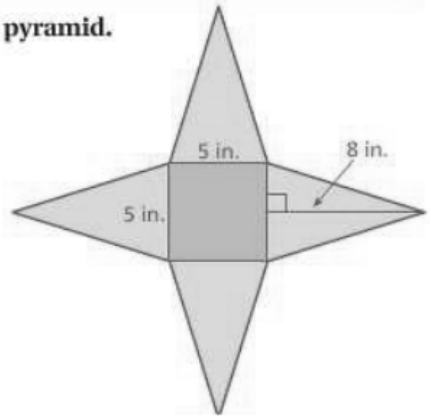
S = area of base + areas of lateral faces

Remember
In a regular polygon, all the sides are congruent and all the angles are congruent.

EXAMPLE 1 Finding the Surface Area of a Square Pyramid

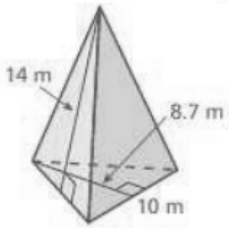
Find the surface area of the regular pyramid.

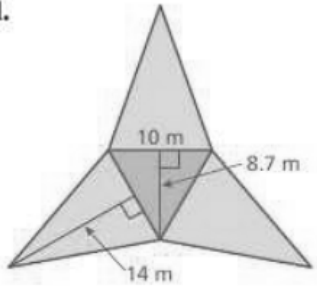




EXAMPLE 2 Finding the Surface Area of a Triangular Pyramid

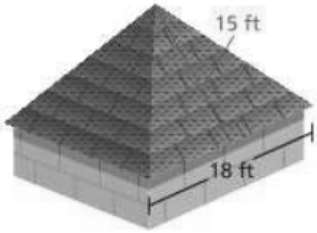
Find the surface area of the regular pyramid.





EXAMPLE 3 Real-Life Application

A roof is shaped like a square pyramid. One bundle of shingles covers 25 square feet. How many bundles should you buy to cover the roof?



9.3 Lesson

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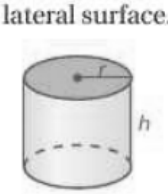
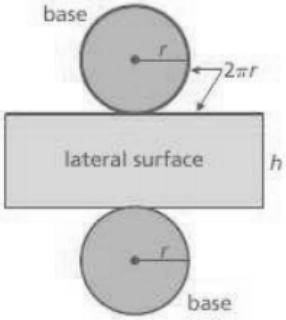
Key Idea

Surface Area of a Cylinder

Words The surface area S of a cylinder is the sum of the areas of the bases and the lateral surface.





Algebra $S = 2\pi r^2 + 2\pi rh$

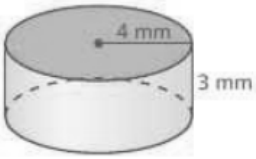
Areas of bases

Area of lateral surface

Remember

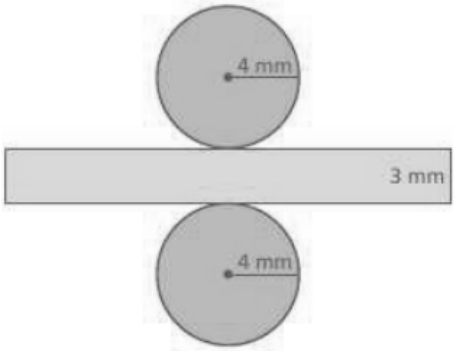
Pi can be approximated as 3.14 or $\frac{22}{7}$.

EXAMPLE 1 Finding the Surface Area of a Cylinder



Find the surface area of the cylinder. Round your answer to the nearest tenth.

Draw a net.



EXAMPLE 2 Finding Surface Area

How much paper is used for the label on the can of peas?

Find the lateral surface area of the cylinder.

EXAMPLE 3 Real-Life Application

You earn $0.01 for recycling the can in Example 2. How much can you expect to earn for recycling the tomato can? Assume that the recycle value is proportional to the surface area.

Find the surface area of each can.

Tomatoes ***Peas***

9.4 Lesson

Check It Out
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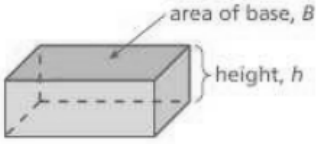
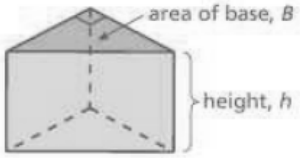
The *volume* of a three-dimensional figure is a measure of the amount of space that it occupies. Volume is measured in cubic units.

Key Idea

Volume of a Prism

Words The volume V of a prism is the product of the area of the base and the height of the prism.





Algebra $V = Bh$

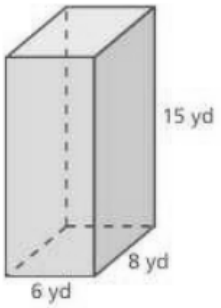
Area of base — Height of prism

Remember

The volume V of a cube with an edge length of s is $V = s^3$.

EXAMPLE 1 Finding the Volume of a Prism

Find the volume of the prism.

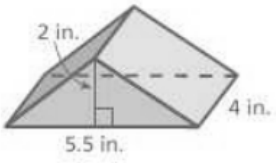


Study Tip

The area of the base of a rectangular prism is the product of the length ℓ and the width w.
You can use $V = \ell wh$ to find the volume of a rectangular prism.

EXAMPLE 2 Finding the Volume of a Prism

Find the volume of the prism.

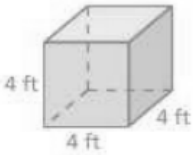


On Your Own

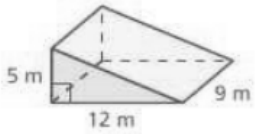
Now You're Ready
Exercises 4–12

Find the volume of the prism.

1.

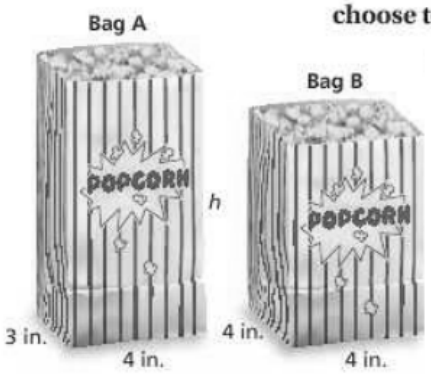


2.



EXAMPLE 3 Real-Life Application

A movie theater designs two bags to hold 96 cubic inches of popcorn. (a) Find the height of each bag. (b) Which bag should the theater choose to reduce the amount of paper needed? Explain.



9.5 Lesson

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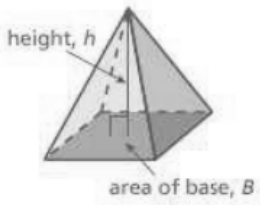
Key Idea

Volume of a Pyramid

Words The volume V of a pyramid is one-third the product of the area of the base and the height of the pyramid.



Algebra $V = \frac{1}{3}Bh$

Area of base

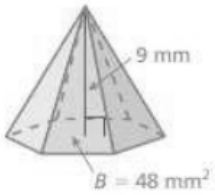
Height of pyramid

Study Tip

The *height* of a pyramid is the perpendicular distance from the base to the vertex.

EXAMPLE 1 Finding the Volume of a Pyramid

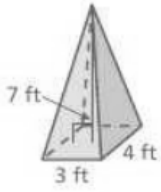
Find the volume of the pyramid.



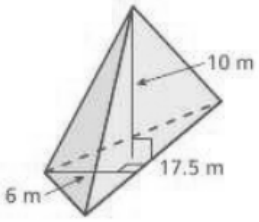
EXAMPLE 2 Finding the Volume of a Pyramid

Find the volume of the pyramid.

a.



b.



Study Tip

The area of the base of a rectangular pyramid is the product of the length ℓ and the width w.

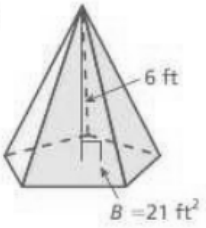
You can use $V = \frac{1}{3}\ell wh$ to find the volume of a rectangular pyramid.

On Your Own

Now You're Ready
Exercises 4–11

Find the volume of the pyramid.

1.



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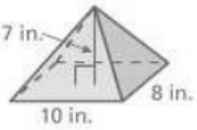
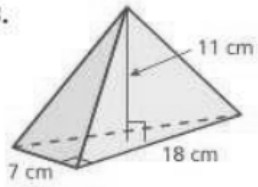


3.



EXAMPLE 3 Real-Life Application

a. The volume of sunscreen in Bottle B is about how many times the volume in Bottle A?

b. Which is the better buy?

a. Use the formula for the volume of a pyramid to estimate the amount of sunscreen in each bottle.

Bottle A ***Bottle B***

Extension 9.5 Cross Sections of Three-Dimensional Figures

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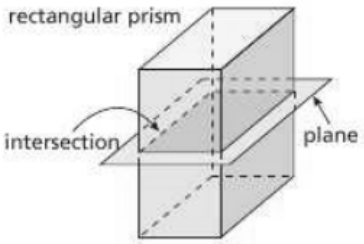
Key Vocabulary
cross section, *p. 388*

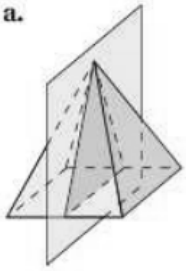
Consider a plane "slicing" through a solid. The intersection of the plane and the solid is a two-dimensional shape called a **cross section**. For example, the diagram shows that the intersection of the plane and the rectangular prism is a rectangle.



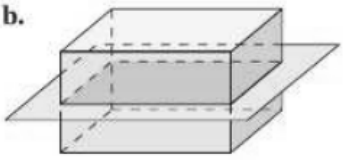
EXAMPLE 1 Describing the Intersection of a Plane and a Solid

Describe the intersection of the plane and the solid.

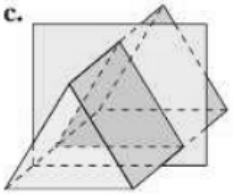
a.



b.



c.



FLORIDA STANDARDS

Geometry

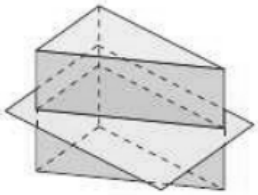
In this extension, you will
- describe the intersections of planes and solids.

Learning Standard
MAFS.7.G.1.3

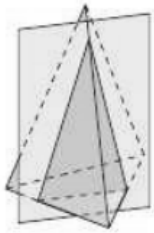
Practice

Describe the intersection of the plane and the solid.

1.



2.



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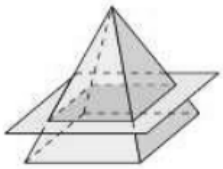

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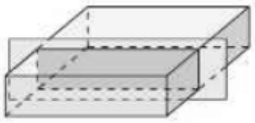

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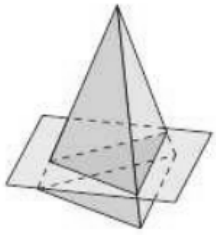
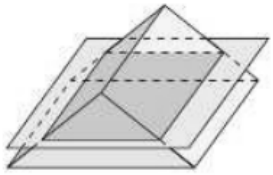
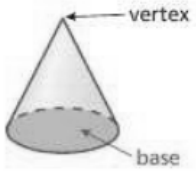


6.



7. REASONING A plane that intersects a prism is parallel to the bases of the prism. Describe the intersection of the plane and the prism.

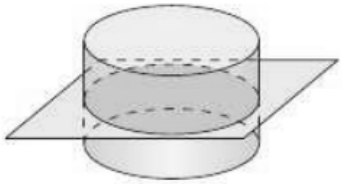
Example 1 shows how a plane intersects a polyhedron. Now consider the intersection of a plane and a solid having a curved surface, such as a cylinder or cone. As shown, a *cone* is a solid that has one circular base and one vertex.



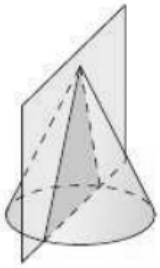
EXAMPLE 2 Describing the Intersection of a Plane and a Solid

Describe the intersection of the plane and the solid.

a.



b.



Math Practice 1

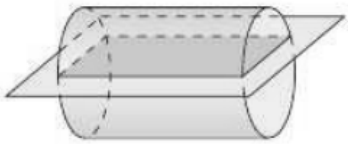
Analyze Givens

What solid is shown? What are you trying to find? Explain.

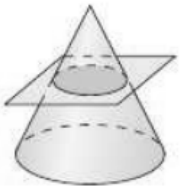
Practice

Describe the intersection of the plane and the solid.

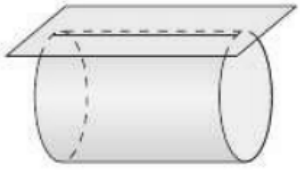
8.



9.



10.



11.

Describe the shape that is formed by the cut made in the food shown.

12.

13.

14.

15. REASONING Explain how a plane can be parallel to the base of a cone and intersect the cone at exactly one point.